Hamburger Mathematics: Introduction to Functions

Dr. Frank's Math Minute:
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http://www.wangeducation.com/mathminute/hamburger.shtml

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Goal:
To teach students about functions, their definition, how to calculate the values of functions for various values of the independent variable, and how functions may arise.

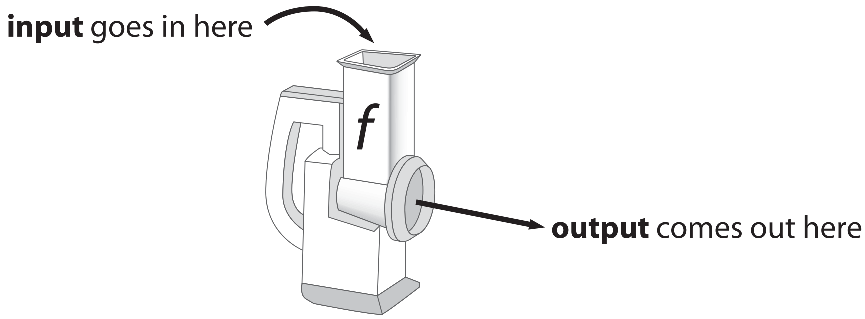
Prerequisites:
Pre-algebra; knowledge of variables: knowing that a letter such as x can stand for a number; knowledge of squaring a number, i.e., $3^2 = 9$

Background:
This worksheet is designed to be used in conjunction with Dr. Frank's Math Minute "Hamburger Mathematics – An Application of Newton's Law of Cooling." On the video, Dr. Frank shows two machines – actually two food processors. One machine is labeled T and one is labeled T' (read "T prime").

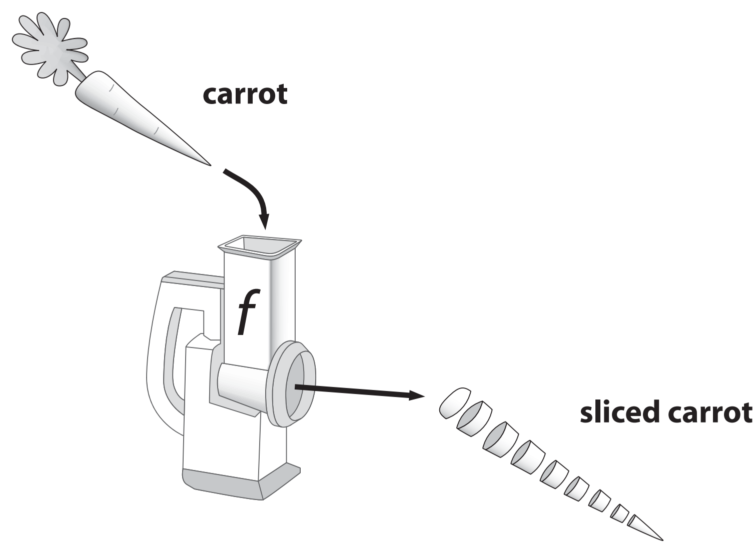
Both of these machines represent what in mathematics are called functions. We will only study the function named T. The function named T' is mathematically related to the function named T but this relationship cannot be explained without knowledge of an area of mathematics called calculus.

The capital letter t, T, stands for Temperature. The machine represents what we call the temperature function or simply the T function.

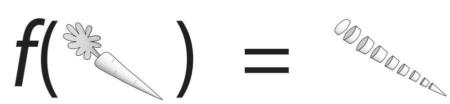
Introduction:
What is a function? **A function is a machine that takes in something called an *input* and uses it to produce something called an *output*.** A food processor can be thought of as a function. The name of the function below is *f*.



For example, a whole carrot may be used as an input. What comes out will be something like sliced carrot.

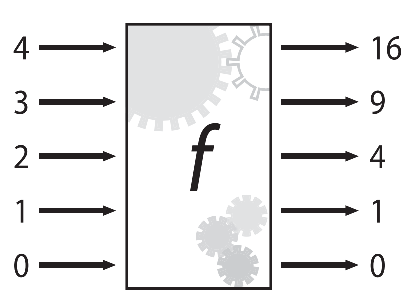


We can express the picture above in the form of an equation by writing …



The function *f* acts upon the input, which we put within parentheses. The output is shown on the other side of the = symbol.

The functions we will study will have numbers as both inputs and outputs. An example of a function machine with numbers as inputs and outputs is shown below.

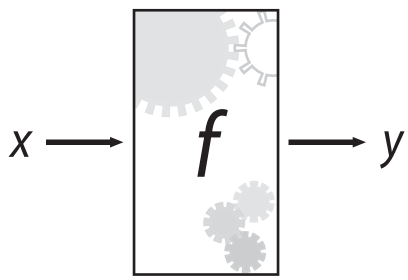


For the previous function …

$$f(4) = 16$$
$$f(3) = 9$$
$$f(2) = 4$$
$$f(1) = 1$$
$$f(0) = 0$$

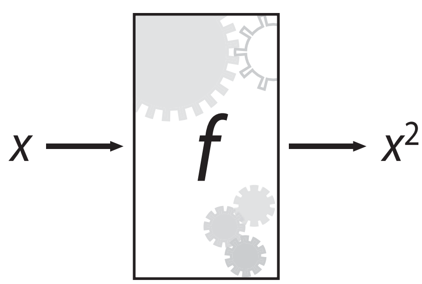
Unfortunately, with such a picture, we can only show what the function does to a limited number of inputs. To describe a function that can have an unlimited number of inputs, we must describe *f* using an equation.

In writing the equation for a function, we usually begin by using the letter *x* to represent the inputs; used in such a way, the letter *x* is said to be the *independent variable*. The output is typically represented by the letter *y* and is said to be the *dependent variable* (simply because its value depends on the value of the independent variable).



$\mathbf{f(x) = y}$

Below, we show a function whose equation is $f(x) = x^2$ or $y = x^2$. This means that an input, *x*, will be squared by the machine and the output would be x^2. x^2, as you know, is simply shorthand for expressing *x* multiplied by itself. For example, 3^2 is equal to 3×3 or 9.

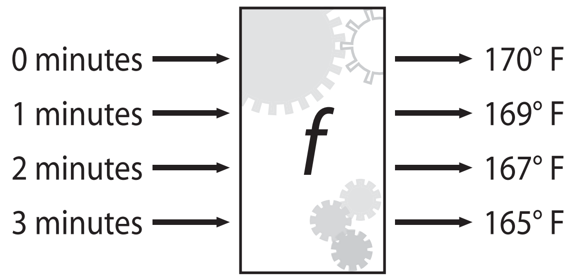


$\mathbf{f(x) = x^2}$

Sometimes functions have some meaning and a name for them is chosen to reflect that meaning. In the case of Dr. Frank's experiment, *T* is the name of the function that describes the Temperature of a hamburger that is cooling. We use the lowercase *t*, standing for time, as the independent variable since the Temperature, *T*, of the hamburger depends on the time, *t*.

Let's work some examples involving the temperature function *T*. The Temperature function tells us the temperature of the hamburger for different times after the hamburger is removed from the grill.

Example 1:
Suppose the Temperature function is shown below.



The Temperature function tells us that the moment the hamburger leaves the grill, it is 170° F. After 1 minute, the hamburger is 169° F. After 2 minutes, the hamburger is 167° F and so forth.

What is the value of T(3 minutes)?

Solution:
The value of T(3 minutes) is the output of the Temperature function when $t = 3$ minutes. The answer is **165° F.**

Usually mathematicians will omit the units in calculating with functions. We keep the units here because they have meaning in our problem. In the remaining examples, we omit the units.

Example 2:
Suppose the Temperature function is given by the equation $T(t) = 170 - 3t$ where t varies from 0 to 25 minutes. What is the value of $T(0)$, $T(1)$, and $T(10)$?

Solution:
The value of T(0) can be found by substituting 0 for the value of t.

$$T(0) = 170 - 3(0)$$
$$= \mathbf{170}$$

The value of a function such as temperature T at $t = 0$ is often referred to as the *initial* value of the function. In our case, this temperature represents the first time the temperature of the hamburger is taken.

Similarly, we compute the values of $T(1)$ and $T(2)$.

$$T(1) = 170 - 3(1) = \mathbf{167}$$
$$T(2) = 170 - 3(2) = \mathbf{164}$$

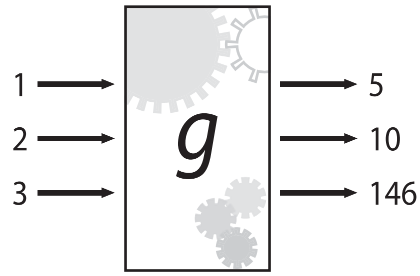
Example 3:
What is the value of T at $t = 0, 1,$ and 2 if the equation of T is given by $T(t) = 3t + 20$?

Solution:

$$T(0) = 3(0) + 20 = \mathbf{20}$$
$$T(1) = 3(1) + 20 = \mathbf{23}$$
$$T(2) = 3(2) + 20 = \mathbf{26}$$

Problem Set

1. The function machine, g, is shown.



a. What is the output of g when the input is a 1?

b. What is the value of g(3)?

2. A hamburger is removed from a grill and immediately the hamburger's temperature begins to fall in accordance to the following temperature equation

$$T(t) = 100 - 4t$$

where T is measured in degrees Fahrenheit and t is measured in minutes.

a. What is the initial temperature of the hamburger?

b. What is the temperature of the hamburger at $t = 10$ minutes?

c. What time after measurements begin is the temperature of the hamburger precisely 0?

3. The sales of a particular product (as a function of its price) is given by the function

$$S(p) = p \times (10 - p)$$

where the price is measured in dollars.

a. What are the total sales when the product is priced at $0 (in other words, the product is free)?

b. What are the sales when the price is $5?

c. What are the sales when the price is $8?

d. What are the sales when the price is $10?

4. Draw a function machine (similar to the one in Example 1) that takes an input, squares it, and then subtracts 1 from the result. Use the following four inputs: 1, 2, 3, and 4.